

2020 Middle School Math Madness Practice Round

1. To the nearest minute, how much time could Robert save on a 60 mile trip if he drove at 70 mph rather than at 50 mph?

ANSWER: 21

$$\frac{60 \text{ miles}}{50 \text{ mph}} - \frac{60 \text{ miles}}{70 \text{ mph}} = 6 \left(\frac{1}{5} - \frac{1}{7} \right) \text{ hour} = \frac{12}{35} \text{ hour}$$

Then

$$\frac{12}{35} \text{ hour} \cdot \frac{60 \text{ minutes}}{\text{hour}} = 20\frac{4}{7} \text{ minutes} \approx 20.57 \text{ minutes.}$$

To the nearest minute, the answer is 21.

2. Solve for x :

$$\frac{x}{57.8} = \frac{25}{17}$$

ANSWER: 85

$$\begin{aligned} \frac{x}{57.8} &= \frac{25}{17} \\ \frac{10x}{2 \cdot 289} &= \frac{25}{17} \\ 5x &= 25 \cdot 17 \\ x &= 5 \cdot 17 = 85. \end{aligned}$$

The answer is 85.

3. Matilda has lots of socks. In fact, she has 4 socks of each of the colors of the rainbow: red, orange, yellow, green, blue, indigo, and purple in the drawer. Furthermore, Matilda likes to wear socks of different colors. If in the dark and unable to distinguish colors, what is the least number of socks she must select to be sure she has two socks of different colors?

ANSWER: 5

In the worst case, Matilda will select 4 socks of the same color. Then the 5th sock will necessarily be of a different color.

4. How many digits does the quotient $123246369/123$ have?

ANSWER: 7

$$\frac{123246369}{123} = 1002003$$

so there are 7 digits in the quotient.

5. Assume that a raindrop's volume is 1 cubic centimeter. In centimeters, what would be the depth of water in a pan whose square base measured 1 meter by 1 meter after 1 hour of rain falls at a rate of 2 raindrops per second on a 10 centimeter by 10 centimeter region?

ANSWER: 72

Use "cc" for "cubic centimeter". Then 2 cc in one second on a 10 cm by 10 cm (100 cm^2) region is $2 \cdot 10 \cdot 10 = 200$ cc per second on a 100 cm by 100 cm (square meter) region. That is

$$\frac{200 \text{ cc}}{1 \text{ second}} \cdot \frac{60 \text{ seconds}}{1 \text{ minute}} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} = 720,000 \text{ cc}$$

in one hour. The base has an area of 10,000 square centimeters so the height of the water is 72 cm.

6. How many three-digit positive integers have exactly two digits that are the same?

ANSWER: 243

We have AAB , ABA , and BAA . In the first two cases $A \neq 0$ and while B can equal 0 it can't equal A , so there are $9 \cdot 1 \cdot 9$ for the first case and $9 \cdot 9 \cdot 1$ for the second case for a total of $81 + 81 = 162$ possible numbers. In the third case $B \neq 0$ and while A can equal zero, it can't equal B , so we have $9 \cdot 9 \cdot 1 = 81$ possible numbers. The final total is $162 + 81 = 243$.

7. Steve writes some numbers on the whiteboard. He starts with the number 1. Then to write the next number, he multiplies the current number by 2, then subtracts 2, then divides by 2 and finally adds 2. This number becomes the current number, Steve writes it on the board, and then he repeats the process. When he finally writes a three-digit number, how many numbers has he written?

ANSWER: 100

If x is the current number, then the operations lead to

$$2x, 2x - 2, (2x - 2)/2 = x - 1, \text{ and } (2x - 2)/2 + 2 = (x - 1) + 2 = x + 1.$$

So the sequence of numbers will be $1, 2, 3, \dots, 100$. So he writes 100 numbers.

8. Given that a, b, c, d, e are distinct whole numbers from 1 to 9 inclusive, along with the equality

$$a^b = c \cdot 100 + d \cdot 10 + e$$

where $c \cdot 100 + d \cdot 10 + e$ is the largest possible value, find the value of

$$a + b + c + d + e.$$

ANSWER: 27

Testing the expression a^b on the left with $a = 2, 3, \dots, 9$, the largest possible 3-digit results are

$$2^9 = 512, 3^6 = 729, 4^4 = 256, 5^4 = 625, 7^3 = 343, 8^3 = 512, 9^3 = 729.$$

Among these $729 = 3^6 = 9^3$ is the largest. Rule out $9^3 = 729$ because digit 9 is used more than once it does not satisfy the requirements. However $3^6 = 729$ does satisfy the requirements. So $a = 3$, $b = 6$, $c = 7$, $d = 2$ and $e = 9$. Then $a + b + c + d + e = 3 + 6 + 7 + 2 + 9 = 27$.